

Differentiation Techniques

Note: $f(x)$ and $g(x)$ are both functions of x , and $f'(x) = \frac{d}{dx} f(x)$, and $g'(x) = \frac{d}{dx} g(x)$

Basic Differentiation

$$\frac{d}{dx} x^n = nx^{n-1}$$

Remember: Take down the power, power minus 1

Product Rule

$$\frac{d}{dx} f(x) \times g(x) = f(x)g'(x) + f'(x)g(x)$$

Remember: Copy first term and differentiate second term 'plus' copy second term and differentiate first term

Example:

$$\begin{aligned}\frac{d}{dx} (2x+1)(x-5) &= (2x+1) \frac{d}{dx} (x-5) + (x-5) \frac{d}{dx} (2x+1) \\ &= (2x+1)(1) + (x-5)(2) \\ &= 4x-9\end{aligned}$$

Quotient Rule

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f(x)g'(x) - f'(x)g(x)}{[g(x)]^2}$$

Remember: Copy bottom and differentiate top minus copy top and differentiate bottom, whole function divided by bottom squared

Example:

$$\begin{aligned}\frac{d}{dx} \frac{2x+1}{4x-5} &= \frac{(4x-5) \frac{d}{dx} (2x+1) - (2x+1) \frac{d}{dx} (4x-5)}{(4x-5)^2} \\ &= \frac{(4x-5)(2) - (2x+1)(4)}{(4x-5)^2} = \frac{-14}{(4x-5)^2}\end{aligned}$$

Trigonometrical Functions

$$\frac{d}{dx} \sin[f(x)] = f'(x) \cos[f(x)]$$

$$\frac{d}{dx} \cos[f(x)] = f'(x)[- \sin f(x)]$$

$$\frac{d}{dx} \tan[f(x)] = f'(x)[\sec^2 f(x)]$$

Example,

$$\begin{aligned}\frac{d}{dx} \sin(1-10x) &= \frac{d}{dx} (1-10x) \cos(1-10x) \\ &= -10 \cos(1-10x) \\ \frac{d}{dx} \cos(12x-1) &= \frac{d}{dx} (12x-1) \times -\sin(12x-1) \\ &= -12x \sin(12x-1)\end{aligned}$$

Exponential Functions

$$\frac{d}{dx} e^{f(x)} = f'(x)e^{f(x)}$$

Example:

$$\begin{aligned}\frac{d}{dx} e^{(2x^2+x)} &= \frac{d}{dx} (2x^2+x) \times e^{(2x^2+x)} \\ &= (4x+1)e^{(2x^2+x)}\end{aligned}$$

Logarithmic Functions

$$\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$$

Example:

$$\begin{aligned}\frac{d}{dx} \ln(3x^2-10) &= \frac{\frac{d}{dx} (3x^2-10)}{3x^2-10} \\ &= \frac{6x}{3x^2-10}\end{aligned}$$

Chain Rule

$$\frac{d}{dx} [f(x)]^n = n f'(x) [f(x)]^{n-1}$$

Remember: Take down the power, power minus 1, multiplied by the differential of the function inside

Example:

$$\begin{aligned}\frac{d}{dx} (2x+1)^{10} &= 10(2x+1)^{10-1} \frac{d}{dx} (2x+1) \\ &= 10(2x+1)^9(2) \\ &= 20(2x+1)^9\end{aligned}$$